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Running coupling of 2-flavor QCD at zero and finite temperature

O. Kaczmarek[1], F. Zantow[2,a]

[1] Fakultät für Physik, Bielefeld University, 33615 Bielefeld, Germany
[2] Physics Department, Brookhaven Natl. Laboratory, Upton, NY-11973, USA

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Abstract. We present lattice studies of the running coupling in 2-flavor QCD. The coupling at zero temperature ($T = 0$) is extracted from Wilson loops while the coupling at finite temperature ($T \neq 0$) is determined from Polyakov loop correlation functions.

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1 Running couplings

The QCD coupling plays an important role at zero temperature and, in particular, at finite temperature in todays discussion of possible signals for the quark–gluon plasma formation in heavy ion experiments [1–3]. We calculate running couplings from lattice studies of the Wilson loop ($T = 0$) [4] and Polyakov loop correlation functions ($T \neq 0$) in 2-flavor QCD ($N_f = 2$) using an improved staggered fermion action with quark mass $m/T = 0.4$ (corresponding to $ma = 0.1$) [5]. Any further details on this study can be found in [4, 6, 7]. Similar studies in quenched QCD are reported in [8–10]. First experiences with the running coupling at finite temperature in 3-flavor QCD are reported in [11].

1.1 Heavy quark potential at $T = 0$

For the determination of the heavy quark potential at zero temperature, $V(r)$, we have used the measurements of large smeared Wilson loops given in [4] ($N_f = 2$ and $ma = 0.1$). To eliminate the divergent self-energy contributions we matched these data for all β-values (different β-values correspond to different values of the lattice spacing a) at large distances to the bosonic string potential,

$$V(r) = -\frac{\pi}{12}\frac{1}{r} + \sigma r \equiv -\frac{4}{3}\frac{\alpha_{\rm str}}{r} + \sigma r \ , \qquad (1)$$

where we already have separated the Casimir factor so that $\alpha_{\rm str} \equiv \pi/16$. In Fig. 1a,b we show our results together with the heavy quark potential from the string picture (dashed line). One can see that the data are well described by (1) at large distances, i.e. $r\sqrt{\sigma} \gtrsim 0.8$, corresponding to $r \gtrsim 0.4$ fm. At these distances we see no major difference between the

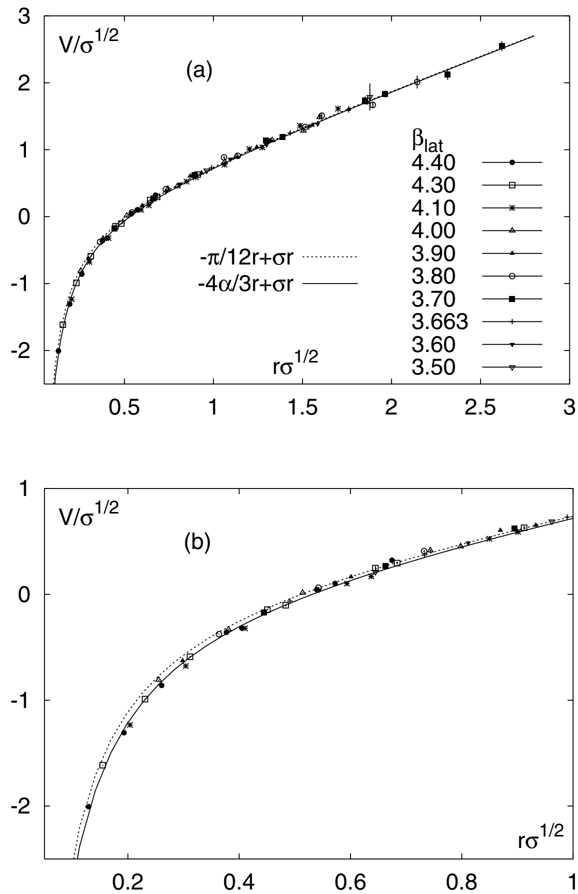

Fig. 1. a The heavy quark potential at $T = 0$ from [4] obtained from 2-flavor QCD lattice simulations with quark masses $ma = 0.1$ for different values of the lattice coupling β. **b** shows an enlargement of the short distance regime. The data are matched to the bosonic string potential (dashed line) at large distances. Included is also the fit to the Cornell form (solid line) given in (3)

[a] e-mail: zantow@quark.phy.bnl.gov

2-flavor QCD potential obtained from Wilson loops and the quenched QCD potential which is well described by the string model already for $r \gtrsim 0.4\,\mathrm{fm}$ [8, 12]. In fact, we also do not see any signal for string breaking in the zero temperature QCD heavy quark potential. This is to some extent due to the fact that the Wilson loop operator used here for the calculation of the $T = 0$ potential has only small overlap with states where string breaking occurs [13]. Moreover, the distances for which we analyze the data for the QCD potential are all below $r \lesssim 1.5\,\mathrm{fm}$ at which string breaking is expected to set in at zero temperature.

1.2 The coupling at $T = 0$

Deviations from the string model and from the pure gauge potential, however, are clearly expected to become apparent in the 2-flavor QCD potential at small distances and may already be seen from the short distance part in Fig. 1. These deviations are expected to arise from an asymptotic weakening of the QCD coupling, i.e. $\alpha = \alpha(r)$, and also is to some extent due to the effect of including dynamical quarks ($N_f \neq 0$), i.e. from leading order perturbation theory one expects

$$\alpha(r) \simeq \frac{1}{8\pi} \frac{1}{\beta_0 \log\left(1/(r\Lambda_{\mathrm{QCD}})\right)} \ , \tag{2}$$

with $\beta_0 = (33 - 2N_f)/(48\pi^2)$ where N_f is the number of flavors. The data in Fig. 1b show a slightly steeper slope at distances below $r\sqrt{\sigma} \simeq 0.5$ compared to the pure gauge potential given in [8] indicating that the QCD coupling gets stronger in the entire distance range analyzed here when including dynamical quarks. To include the effect of a stronger Coulombic part in the QCD potential we test the Cornell parameterization,

$$\frac{V(r)}{\sqrt{\sigma}} = -\frac{4}{3}\frac{\alpha}{r\sqrt{\sigma}} + r\sqrt{\sigma} \ , \tag{3}$$

with a free parameter α. From a best fit analysis of (3) to the data ranging from $0.2 \lesssim r\sqrt{\sigma} \lesssim 2.6$ we find $\alpha = 0.212(3)$. This already may indicate that the logarithmic weakening of the coupling with decreasing distance will not too strongly influence the properties of the QCD potential at these distances, i.e. at $r \gtrsim 0.1\,\mathrm{fm}$. However, the value of α is moderately larger than $\alpha_{\mathrm{str}} \simeq 0.196$ introduced above. To compare the relative size of α in full QCD to α calculated in the quenched theory we again have performed a best fit analysis of the quenched zero temperature potential given in [8] using the ansatz given in (3) and a similar distance range. Here we find $\alpha_{\mathrm{quenched}} = 0.195(1)$ which is again smaller than the value for the QCD coupling but quite comparable to α_{str}.

When approaching the short distance perturbative regime a Cornell ansatz will overestimate the value of the coupling due to the perturbative logarithmic weakening of the latter, $\alpha_{\mathrm{QCD}} = \alpha_{\mathrm{QCD}}(r)$. To analyze the short distance properties of the QCD potential and the coupling in more detail, i.e. at $r \lesssim 0.4\,\mathrm{fm}$, and to firmly establish here the onset of its perturbative weakening with decreasing distance,

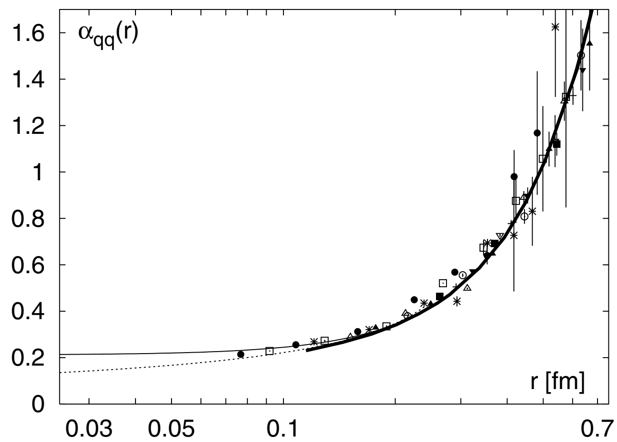

Fig. 2. The short distance part of the running coupling $\alpha_{qq}(r)$ in 2-flavor QCD at zero temperature defined in (4) as function of the distance r (in physical units). The symbols for the different β-values are chosen as indicated in Fig. 1a. The lines are discussed in the text

it is customary to do so using non-perturbative definitions of running couplings. Following recent discussions on the running of the QCD coupling [8, 14, 15], it appears most convenient to study the QCD force, i.e. $\mathrm{d}V(r)/\mathrm{d}r$, rather than the QCD potential. In this case one defines the QCD coupling in the so-called qq-scheme,

$$\alpha_{qq}(r) \equiv \frac{3}{4} r^2 \frac{\mathrm{d}V(r)}{\mathrm{d}r} \ . \tag{4}$$

In this scheme any undetermined constant contribution to the heavy quark potential cancels out. Moreover, the large distance, non-perturbative confinement contribution to $\alpha_{qq}(r)$ is positive and allows for a smooth matching of the perturbative short distance coupling to the non-perturbative large distance confinement signal.

Our results for $\alpha_{qq}(r)$ as a function of distance in physical units for 2-flavor QCD are summarized in Fig. 2. The symbols for the different β-values are chosen as in Fig. 1a. We again show in that figure the corresponding line for the Cornell fit (solid line). At large distances, $r \gtrsim 0.4\,\mathrm{fm}$, the data clearly mimic the non-perturbative confinement part of the QCD force, $\alpha_{qq}(r) \simeq 3r^2\sigma/4$. We also compare our data to the recent high statistics calculation in pure gauge theory (thick solid line). These data are available for $r \gtrsim 0.1\,\mathrm{fm}$ and within the statistics of the QCD data no significant differences could be identified between the QCD and pure gauge data for $r \gtrsim 0.4\,\mathrm{fm}$. At smaller distances ($r \lesssim 0.4\,\mathrm{fm}$), however, the data show some enhancement compared to the coupling in quenched QCD. The data below $0.1\,\mathrm{fm}$, moreover, fall below the large distance Cornell fit. This may indicate the logarithmic weakening of the coupling. At smaller distances than $0.1\,\mathrm{fm}$ we therefore expect the QCD potential to be influenced by the weakening of the coupling and $\alpha_{qq}(r)$ will approach values clearly smaller than α deduced from the Cornell ansatz. Unfortunately we can, at present, not go to smaller distances to clearly demonstrate this behavior with our data in 2-flavor QCD. Moreover, at small distances cut-off effects may also influence our analysis of the coupling and more detailed studies are required here. In earlier studies of the coupling in pure gauge theory [8,9,15] it has, however, been shown that the perturbative logarithmic weakening becomes already important at distances smaller than 0.2 fm and contact with perturbation theory could be established.

1.3 The running coupling at $T \neq 0$

We extend here our studies of the coupling at zero temperature to finite temperature below and above deconfinement following the conceptual approach given in [9,10]. In this case the appropriate observable is the color singlet quark anti-quark free energy and its derivative. We use the perturbative short and large distance relation from one gluon exchange [16–18], i.e. in the limit $r\Lambda_{\rm QCD} \ll 1$ zero temperature perturbation theory suggests

$$F_1(r,T) \simeq -\frac{4}{3}\frac{\alpha(r)}{r} \;, \qquad (5)$$

while high temperature perturbation theory, i.e. $rT \gg 1$ and T well above $T_{\rm c}$, yields

$$F_1(r,T) \simeq -\frac{4}{3}\frac{\alpha(T)}{r}\,{\rm e}^{-m_{\rm D}(T)r} \;. \qquad (6)$$

In both relations we have neglected any constant contributions to the free energies which, in particular, at high temperatures will dominate the large distance behavior of the free energies. Moreover, we already anticipated here the running of the couplings with the expected dominant scales r and T in both limits. At finite temperature we define the running coupling in analogy to $T=0$ as,

$$\alpha_{qq}(r,T) \equiv \frac{3}{4}r^2\frac{{\rm d}F_1(r,T)}{{\rm d}r} \;. \qquad (7)$$

With this definition any undetermined constant contributions to the free energies are eliminated and the coupling defined here at finite temperature will recover the coupling at zero temperature defined in (4) in the limit of small distances. Therefore $\alpha_{qq}(r,T)$ will show the (zero temperature) weakening in the short distance perturbative regime. In the large distance limit, however, the coupling will be dominated by (6) and will again be suppressed by color screening, $\alpha_{qq}(r,T) \sim \exp(-m_{\rm D}(T)r)$, $rT \gg 1$. It thus will exhibit a maximum at some intermediate distance.

Lattice results for $\alpha_{qq}(r,T)$ calculated in this way are shown in Fig. 3 and are compared to the coupling at zero temperature discussed already in Sect. 1.2. Here the thin solid line corresponds to the coupling in the Cornell ansatz given in (3). We again show in this figure the results from $SU(3)$-lattice (thick line) and perturbative (dashed line) calculations at zero temperature from [8,15]. The strong r-dependence of the running coupling near $T_{\rm c}$ observed already in pure gauge theory [9,10] is also visible in 2-flavor QCD. Although our data for 2-flavor QCD do not allow for a detailed quantitative analysis of the running coupling at smaller distances, the qualitative behavior is

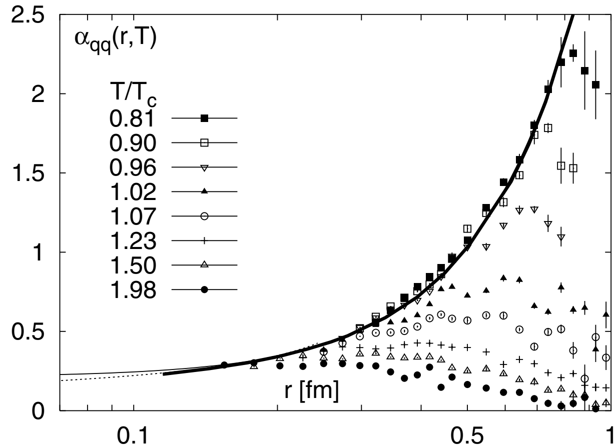

Fig. 3. The running coupling in the qq-scheme defined in (7) calculated from derivatives of the color singlet free energies with respect to r at several temperatures as function of distance below and above deconfinement. We also show the corresponding coupling at zero temperature (solid line) from (3) and compare the results again to the results in pure gauge theory (thick solid and dashed lines) [8,15]

in quite good agreement with the recent quenched results. At large distances the running coupling shows a strong temperature dependence which sets in at shorter distances with increasing temperature. For small temperatures, $T \lesssim 1.02T_{\rm c}$, the coupling $\alpha_{qq}(r,T)$ already coincides with $\alpha_{qq}(r)$ at distance $r \simeq 0.4$ fm and clearly mimics here also the confinement part of $\alpha_{qq}(r)$. This is also apparent in quenched QCD [9]. Remnants of the confinement part of the QCD force may survive the deconfinement transition. A clear separation of the different effects usually described by the concepts of color screening ($T \gtrsim T_{\rm c}$) and effects commonly described by the concept of string breaking ($T \lesssim T_{\rm c}$) is difficult to establish at temperatures in the close vicinity of the confinement deconfinement cross over.

We also analyzed the temperature dependence of the maximal value that $\alpha_{qq}(r,T)$ at fixed temperature exhibits at a certain distance, $r_{\rm max}$, i.e. we identify a temperature dependent coupling, $\tilde\alpha_{qq}(T)$, defined as

$$\tilde\alpha_{qq}(T) \equiv \alpha_{qq}(r_{\rm max},T) \;. \qquad (8)$$

Values for $\tilde\alpha_{qq}(T)$ are also available in pure gauge theory [9] at temperatures above deconfinement [1]. Our results for $\tilde\alpha_{qq}(T)$ in 2-flavor QCD and pure gauge theory are shown in Fig. 4 as function of temperature, $T/T_{\rm c}$. At temperatures above deconfinement we cannot identify significant differences between the data from pure gauge and 2-flavor QCD[2]. Only at temperatures quite close but above the phase transition small differences between full and quenched QCD become visible in $\tilde\alpha_{qq}(T)$. Nonetheless, the value of $\tilde\alpha_{qq}(T)$ drops from about 0.5 at temperatures only moderately larger than the transition temperature, $T \gtrsim 1.2T_{\rm c}$, to a

[1] In pure gauge theory $r_{\rm max}$ and $\tilde\alpha_{qq}(T)$ would be infinite below $T_{\rm c}$.

[2] Note, however, the change in temperature scale from $T_{\rm c} \simeq 200$ MeV in full to $T_{\rm c} \simeq 270$ MeV in quenched QCD.

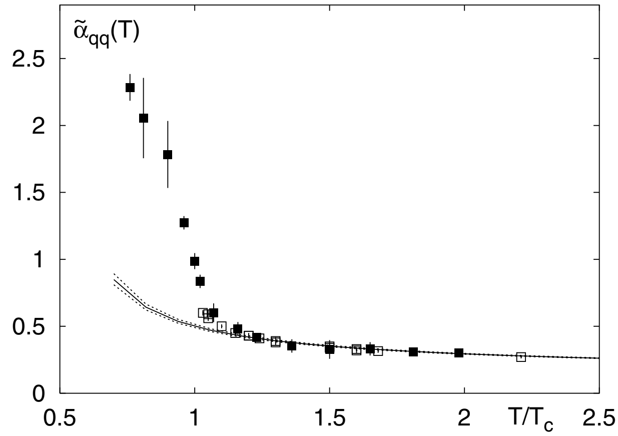

Fig. 4. The size of the maximum, $\tilde{\alpha}_{qq}(T)$, defined in (8), as function of temperature in 2-flavor QCD (filled symbols) and pure gauge theory (open symbols) from [9]. The lines are explained in the text

value of about 0.3 at $2T_c$. This change in $\tilde{\alpha}_{qq}(T)$ with temperature calculated in 2-flavor QCD does not appear to be too dramatic and can indeed be described by the 2-loop perturbative coupling assuming vanishing quark masses. Due to the ambiguity in setting the scale in perturbation theory we performed a best fit analysis to fix this scale for the entire temperature range, $1.2 \lesssim T/T_c \lesssim 2$. We find $T_c/\Lambda = 0.43(1)$ with $\mu = 2\pi T$. This is shown by the solid line (fit) in Fig. 4 including the error band (dotted lines).

At temperatures in the vicinity and below the phase transition temperature, $T \lesssim 1.2T_c$, the behavior of $\tilde{\alpha}_{qq}(T)$ is, however, quite different from the perturbative logarithmic change with temperature. The values for $\tilde{\alpha}_{qq}(T)$ rapidly grow here with decreasing temperature and approach non-perturbative large values. This again shows that $\alpha_{qq}(r,T)$ mimics the confinement part of the zero temperature force still at relatively large distances and that this behavior sets in already at temperatures close but above deconfinement.

2 Summary

Our analysis of the heavy quark potential and coupling in 2-flavor QCD at $T = 0$ shows that deviations from the string picture set in at $r \lesssim 0.4$ fm. At distances smaller than 0.3 fm also deviations from $V(r)$ obtained from Wilson loops in quenched QCD [8] become apparent. The logarithmic running of the coupling will become a dominant feature in $V(r)$ only for $r \lesssim 0.1$ fm. We demonstrated that the QCD coupling at finite temperature indeed runs with distance and coincides with the zero temperature running coupling at sufficiently small distances. Remnants of the confinement part of the QCD force may survive the deconfinement transition and could play an important role for the discussion of non-perturbative aspects of quark anti-quark interactions at temperatures moderately above T_c. A clear separation of the different effects usually described by color screening ($T \gtrsim T_c$) and effects commonly attributed to string breaking ($T \lesssim T_c$) is difficult to establish at temperatures in the close vicinity of the confinement deconfinement cross over. Similar findings were recently reported in quenched QCD [9,19]. Further details on our study can be found in [7,20].

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